XXIX

NONDIMENSIONAL PARAMETER ANALYSIS

The maximum pressure a container will withstand is a function of the material fatigue strength, the amount of prestress, the number of components N, and the wall ratios k_n. To determine the function dependence on these variables and to determine the best designs, a nondimensional analysis is now presented. The calculations for the analysis of each design were programmed on Battelle's CDC 3400 computer.

Multiring Container

Static Shear Strength Analysis

Although a fatigue criterion of failure has been chosen it is illustrative to review an analysis based upon static shear strength for ductile materials first conducted by Manning[23]. The method outlined here differs from that of Manning and is more straightforward. In this analysis the optimum design is found such that each component of the same material has the same value of maximum shear stress S under the pressure load p. The given information is $p_o = p$, $p_N = 0$, and K. The unknowns are the interface pressures p_n, (N-1) in number; the k_n, N in number and S. The total unknowns are 2N. There are N equations resulting from Equation (15) and having the form

$$S = (p_{n-1} - p_n)\frac{k_n^2}{k_n^2 - 1}, \; n = 1, 2, \ldots, N \qquad (26)$$

There is the equation, $K = k_1 k_2 \ldots k_n$, that relates the k_n and K. Also N-1 equations can be formulated from the requirement that S be a minimum, i.e.,

$$\frac{\partial S}{\partial k_n} = 0, \; n = 1, 2, \ldots, N-1 \qquad (27)$$

(There are not N equations in the Form (27) because there is one equation relating the k_n.) Thus, there are also 2N equations which can be solved for the 2N unknowns. The solution gives

$$p_n = p_{n-1} - \frac{(k_n^2 - 1)}{k_n^2} S, \; n = 1, 2, \ldots, N-1 \qquad (28)$$

$$k_1 = k_2 = \ldots = k_N \qquad (29)$$

$$S = \frac{p}{N} \frac{K^{2/N}}{(K^{2/N} - 1)} \qquad (30)$$

The residual pressures q_n and the required interferences for the shrink-fit assembly have yet to be found. The radial stress σ_{rn} at the radius r_n resulting from the bore pressure p is given by Equation (13a) with K replacing k_n, p replacing p_{n-1}, r_N replacing r_n, r_n replacing r, and $p_n = p_N = 0$. σ_{rn} becomes:

$$\sigma_{rn} = \frac{p}{K^2-1} (1 - k_{n+1}{}^2 \, k_{n+2}{}^2 \ldots k_N{}^2) \tag{31}$$

The pressure p_n is the sum of q_n and $(-\sigma_{rn})$. Therefore,

$$q_n = p_n - (-\sigma_{rn}) \tag{32}$$

The interference as manufactured, Δ_n at r_n, is given by

$$\frac{\Delta_n}{r_n} = \frac{-u_n(r_n)}{r_n} + \frac{u_{n+1}(r_n)}{r_n} \tag{33}$$

where

$u_n(r_n)$ = radial deformation at r_n of cylinder N due to the residual pressure q_n at r_n and the residual pressure q_{n-1} at r_{n-1}.

and

$u_{n+1}(r_n)$ = radial deformation at r_n of cylinder n+1 due to the residual pressure q_n at r_n and the residual pressure q_{n+1} at r_{n+1}.

Substituting the Expressions (32) for q_n into Expressions (14a) for the u_n and substituting the results into Equation (33), we find that Δ_n/r_n reduces to:

$$\frac{\Delta_n}{r_n} = \frac{2p}{NE} \tag{34}$$

The result p/2S given by Equation (30) is plotted in Figure 43 for various N. The limit curve is given by

$$\left(\frac{p}{2S}\right)_{limit} = \frac{K^2-1}{K^2} \tag{35}$$

at which limit the minimum shear stress becomes equal to -S at the bore in the inner cylinder.

Figure 43 has been obtained under the assumption that $\frac{\sigma_\theta - \sigma_r}{2}$ always gives the maximum shear stress. As pointed out by Berman, the maximum shear stress in a closed-end container* is given by $\frac{\sigma_z - \sigma_r}{2}$ when $\sigma_z > \sigma_\theta$.(42) Therefore, it is important to know the limit to $\frac{p}{2S}$ for which σ_z becomes equal to σ_θ. σ_z is given by

*Containers for hydrostatic extrusion generally are not closed-end containers. The effect of axial stress is included here for completeness.